

## Lecture 13: Momentum and momentum conservation

Our understanding of mechanics is becoming more sophisticated.

First, we now understand quite a bit about Newton's laws:

1. If  $\sum \vec{F} = 0$ , then  $\vec{v} = \text{constant}$ .
2.  $\sum \vec{F} = m\vec{a}$
3. To every action there is an equal and opposite reaction.

Second, during the last three lectures we have developed an understanding of the work-energy theorem,

$$W_e = \Delta KE + \Delta PE + E_{dissipated} \quad (1)$$

We also saw that if there is no external force and if there is no energy dissipated, then the mechanical energy is conserved, ie  $\Delta PE + \Delta KE = 0$ . We shall return to the work-energy theorem (Eq. (1)) later when we go through thermal physics.

### A new concept: Momentum

We have used velocity a lot in our analysis of kinematics. However the velocity of an object does not tell us the whole effect of that object hitting us! It is intuitively reasonable and turns out to be correct that the product of the velocity of an object and its mass provides a good measure of the impact of a projectile on a target. But wait a minute, we already have used the kinetic energy,  $mv^2/2$ , doesn't that tell us the effect we are looking for? Well yes, it tells us about energy as seen from the work-energy theorem, however the product  $m\vec{v}$  is a vector and so is dependent on the direction of the projectile, which we know has to be important. Momentum is defined to be  $\vec{p} = m\vec{v}$  (note Eq. 6.1 of the text is wrong) and now we are going to see why it is an important new quantity in analysing problems in mechanics.

Recall that the acceleration of an object is,

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \quad (2)$$

Now if we multiply both sides of this equation by the mass, we have,

$$m\vec{a} = m \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta m\vec{v}}{\Delta t} \quad (3)$$

Now we can use Newton's second law to replace  $m\vec{a}$  by  $\sum \vec{F}$ , and we can use our definition of momentum  $\vec{p} = m\vec{v}$  to write this as,

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} \quad (4)$$

This equation states that the force is equal to the change in momentum per unit time. Also if we know the change in momentum, we can find the average force. We define the impulse  $\vec{I} = \vec{p}_f - \vec{p}_i$ , so that

$$\vec{I} = \sum_i \vec{F}_i \Delta t \rightarrow \vec{F} \Delta t \quad (5)$$

Note that the work (=change in energy) is the dot product of the external force and the displacement while the impulse(=change in momentum) is the scalar product of the external force and the time interval. The last equation uses the fact that we can sum up the external forces as vectors to get one total, net, force  $\vec{F}$ .

### **An important special case - No external force**

*When there is no external force the momentum is conserved so that the final momentum is the same as the initial momentum, ie*

$$\vec{p}_f = \vec{p}_i \quad (6)$$

This is used a lot in the analysis of collisions, particularly in colliders such as Fermilab and the NSCL. Note that if two masses collide, we have to sum up the initial momentum of the two masses and equate it to the sum of the final momenta of the two masses.

*Examples:* Consider conservation of energy and conservation of momentum when:

- (i) Bouncing a superball on the floor.
- (ii) Dropping a hackysack onto the floor.